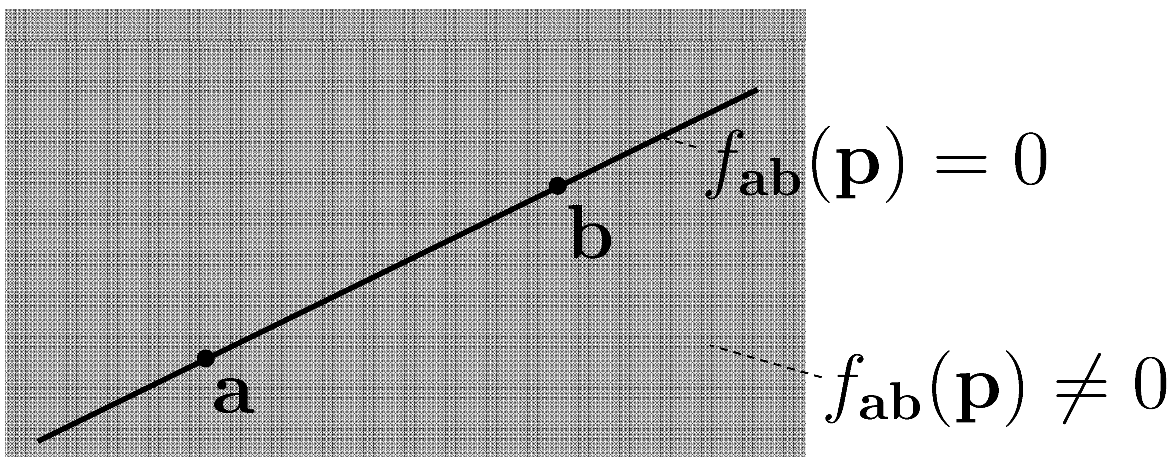
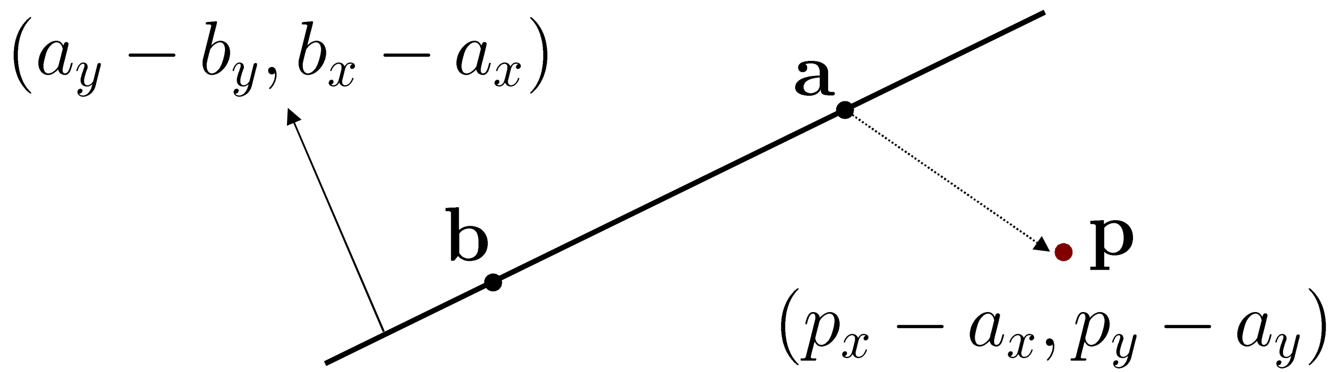
Implicit 2D Lines

- Given two 2D points **a**, **b**
- Define function $f_{\mathbf{ab}}(\mathbf{p})$ such that $f_{\mathbf{ab}}(\mathbf{p}) = 0$ if **p** lies on the line defined by **a**, **b**



Implicit 2D Lines

- Point **p** lies on the line, if **p-a** is perpendicular to the normal of the line

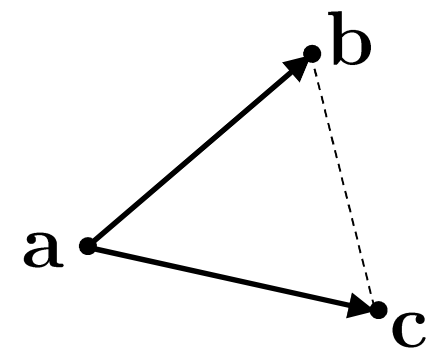


- Use dot product to determine on which side of the line **p** lies. If f(**p**)>0, **p** is on same side as normal, if f(**p**)<0 **p** is on opposite side. If dot product is 0, **p** lies on the line.

$$f_{\mathbf{ab}}(\mathbf{p}) = (a_y - b_y, b_x - a_x) \cdot (p_x - a_x, p_y - a_y)$$

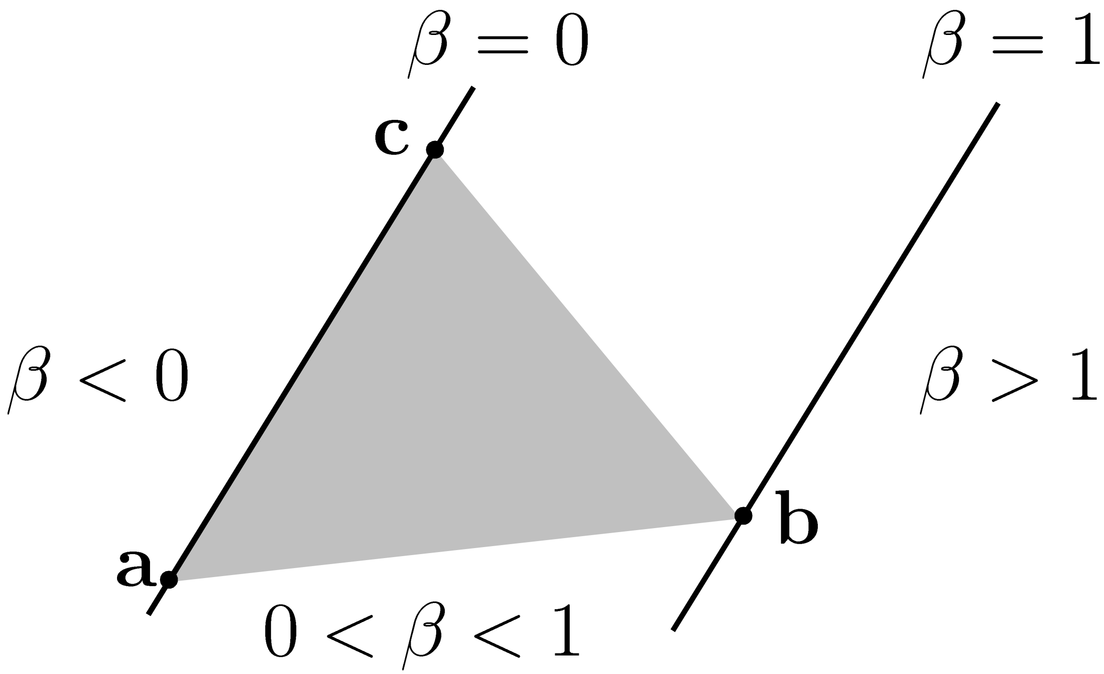
Barycentric Coordinates

- Coordinates for 2D plane defined by triangle vertices ***a, b, c***
- Any point ***p*** in the plane defined by ***a, b, c*** is
  $\boldsymbol{p} = a + \beta(b - a) + \gamma(c - a)$
  $= (1 - \beta - \gamma)\, a + \beta b + \gamma c$
- We define $\alpha = 1 - \beta - \gamma$
  $\Rightarrow \boldsymbol{p} = \alpha a + \beta b + \gamma c$
- α, β, γ are called **barycentric** coordinates
- Works in 2D and in 3D
- If we imagine masses equal to α, β, γ attached to the vertices of the triangle, the center of mass (the barycenter) is then ***p***. This is the origin of the term "barycentric" (introduced 1827 by Möbius)



Barycentric Coordinates

$$\mathbf{p} \;=\; \mathbf{a} + \beta(\mathbf{b}-\mathbf{a}) + \gamma(\mathbf{c}-\mathbf{a})$$



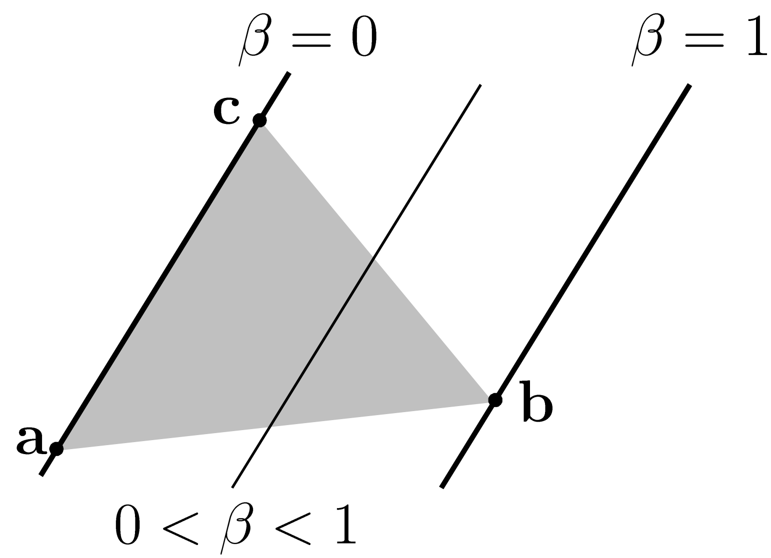
- ***p*** is inside the triangle if $0 < \alpha, \beta, \gamma < 1$

Barycentric Coordinates

- Problem: Given point ***p***, find its barycentric coordinates
- Use equation for implicit lines

$$\beta(\mathbf{p}) = \frac{f_{\mathbf{ac}}(\mathbf{p})}{f_{\mathbf{ac}}(\mathbf{b})}$$

$$\gamma(\mathbf{p}) = \frac{f_{\mathbf{ab}}(\mathbf{p})}{f_{\mathbf{ab}}(\mathbf{c})}$$



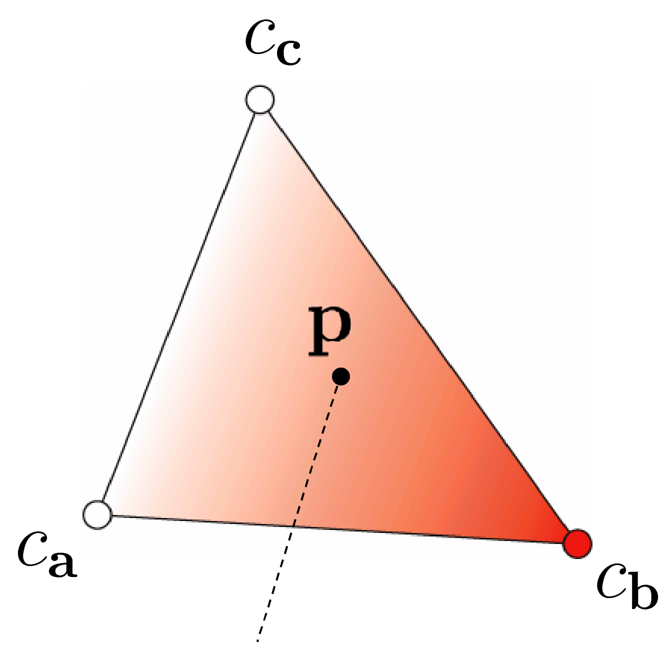
- Division by zero if triangle is degenerate

$$\alpha = 1 - \beta - \gamma$$

$$0 < \beta < 1$$

Barycentric Interpolation

- Interpolate values across triangles, e.g., colors



- Linear interpolation on triangles

$$c(\mathbf{p}) = \alpha(\mathbf{p})c_{\mathbf{a}} + \beta(\mathbf{p})c_{\mathbf{b}} + \gamma(\mathbf{p})c_{\mathbf{c}}$$